4th Year Civil – Structures

2015-2016

Foundation Design

(7)

Elastic Analysis of Shallow Foundations

(Solved examples)

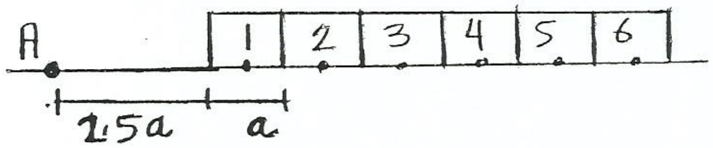
ملاحظات :-

1) اذا طلب في المسألة Maximum Bending Moment ، يتم حسابه كالآتي :-

• بعد إيجاد قيم (P_S) ، يتم التعويض بقيم (P_{S1}) في معادلات $(M-P_S)$ ، وحساب (M) أسفل كل Node ، وأكبر قيمة لـ(M) تكون هي Max. Bending Moment .

2) إذا كان المطلوب في المسألة ، حساب الهبوط لنقطة خارج القاعدة

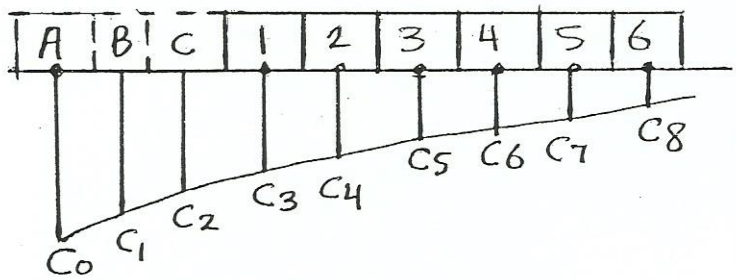
EX1-



• بعد حساب $P_{S1}, P_{S2}, P_{S3}, \ldots$ يتم إيجاد Δ_A كالتالي :-

أ) يتم رسم imaginary elements حتى نصل لنقطة (A) ،

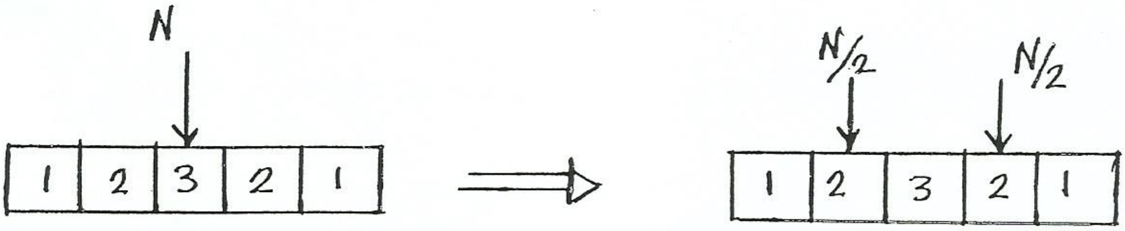
ب) يتم التعويض في معادلات $(P_S-\Delta)$ كالآتي :-



$$\Delta_A = \frac{a}{E_S}\Big[C_0 \times zero + C_1 \times zero + C_2 \times zero + C_3 \times P_{S_1} + C_4 P_{S_2} + C_5 P_{S_3} + C_6 P_{S_4} + C_7 P_{S_5} + C_8 P_{S_6}\Big]$$

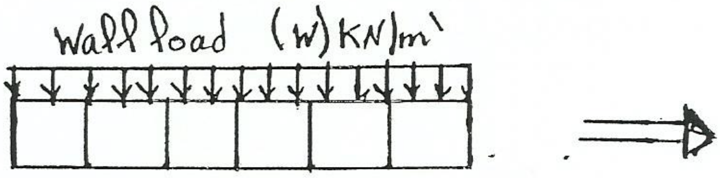
* ويلاحظ أن نقطة (A) تكون في منتصف element ، وأن الإجهاد أسفل الـ elements (A,B,C) تساوي zero.

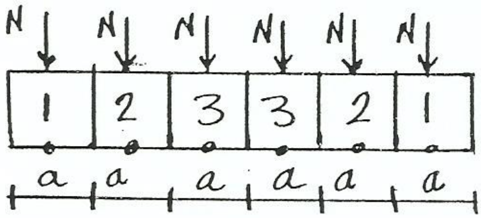
٣)



• إذا كان الحمل معطى في المسألة يؤثر في منتصف القاعدة، يتم تحويله إلى حملين $N/2$ ويؤثروا عند أقرب نقطتي تماثل،
* (منتصف element 2) في المثال السابق.

٤) إذا كان الحمل المعطى في المسألة عبارة عن wall load، يتم التعامل معه كالآتي:-



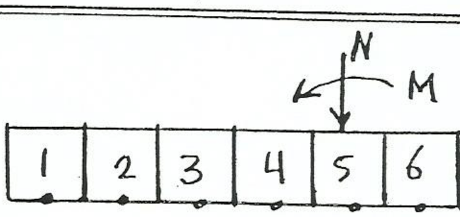


*يتم تحويل حمل الحائط (W) إلى حمل مركز في منتصف كل element يساوي (N) حيث:-

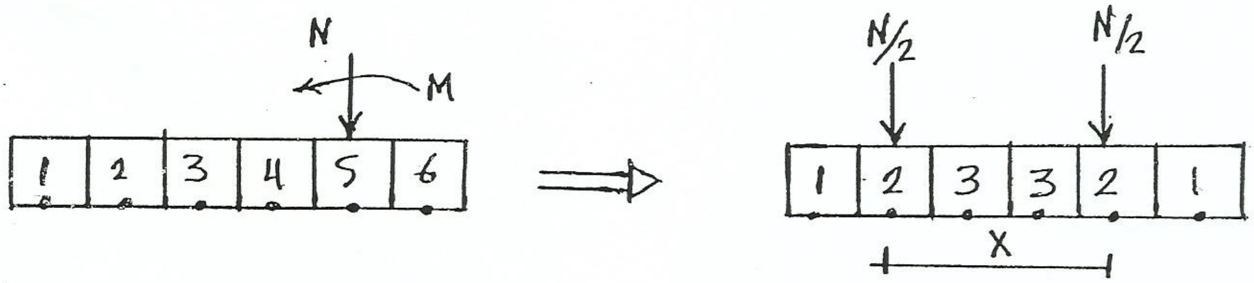
$$N = W * a$$

*ويتم التعامل مع المسألة كقاعدة متماثلة.

٥)



• إذا كانت المسألة عبارة عن قاعدة يؤثر عليها single load and Moment، يتم التعامل معها كالآتي:-



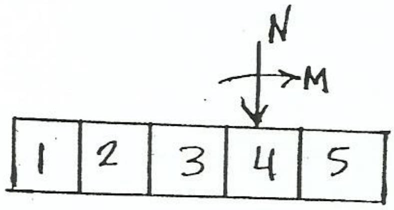
* يتم تحويل المسألة إلى قاعدة متماثلة ، عن طريق تحويل ال Moment و load إلى حملين متساويين متماثلين ، ولتحقيق ذلك يجب تحقق الشروط التالية :-

1) $M = \frac{N}{2} * X$

2) اتجاه دوران (N) حول محور القاعدة ، عكس اتجاه دوران M.

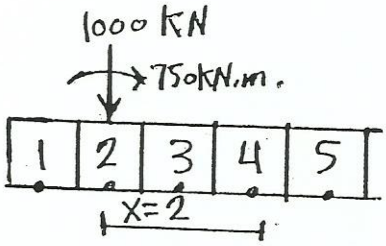
<u>لاحظ الآتي :-</u>

1)



← لا يمكن تحويلها إلى قاعدة متماثلة.

2)



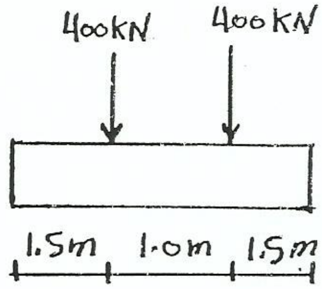
$* \frac{N}{2} = 500$

$* \frac{N}{2} * X = 500 * 2 = 1000 \text{ KN.m.} \neq M$

∴ لا يمكن تحويلها لقاعدة متماثلة.

Example ①:-

- The R.C. footing shown in figure is 1.5 x 4.0m, with thickness = 60cm, and carries two columns 1.0m center to center. The footing is divided into 4 elements.

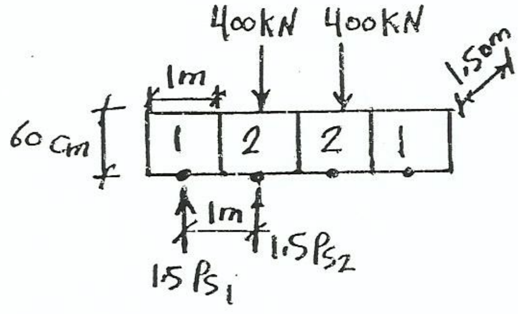


It is required to calculate and draw the contact stress distribution and the settlement diagram below the footing, in the following two cases:-

1) The soil is modelled based on winkler assumptions
   ($K_{so} = 6000\ kN/m^3$)

2) The soil is elastic, homogeneous, isotropic and semi-infinite;
   ($C_o = 1.33$, $C_1 = 0.52$, $C_2 = 0.3$, $C_3 = 0.2$)
   $E_c = 2000\ kN/cm^2$, $E_s = 4000\ kN/m^2$

Solution:-



(A) M-Ps eqs:-

$$M_1 = 0 \quad \text{------} \quad (a)$$

$$M_2 = 1.5\, P_{s1} \quad \text{------} \quad (b)$$

(B) M-Δ eqs:-

$$\frac{6 E_c I}{a^2} = \frac{6 \times 2 \times 10^7 \times 1.5 \times (0.6)^3 / 12}{(1)^2} = 3.24 \times 10^6 \text{ KN}$$

- Applying 4-Moment equation at Point (2)

$$M_1 + 4M_2 + M_2 = 3.24 \times 10^6 \left(-\Delta_1 + 2\Delta_2 - \Delta_2\right)$$

$$\therefore M_1 + 5M_2 = 3.24 \times 10^6 \left(-\Delta_1 + \Delta_2\right) \quad \text{------} \quad (1)$$

(C) Δ-Ps eqs:-

(1) Winkler assumption:-

$$\Delta_1 = \frac{P_{s1}}{6000} \quad \text{------} \quad (I)$$

$$\Delta_2 = \frac{P_{s2}}{6000} \quad \text{------} \quad (II)$$

Final eqs:-

Substituting eqs (a, b), (I, II) in eq. (1)

$$5 \times \underbrace{(1.50\, P_{s1})}_{M_2} = \frac{3.24 \times 10^6}{6000} \left(-P_{s1} + P_{s2}\right)$$

$$\underline{547.5\, P_{s1} - 540\, P_{s2} = 0} \quad \text{------} \quad (1)$$

$\therefore \sum F_y = 0$

$\therefore \quad 2\,(1.5 P_{s1} + 1.5 P_{s2}) = 800$

$$\therefore \quad \underline{P_{s1} + P_{s2} = 266.67} \quad \text{------} \quad (2)$$

Solving eqs (1), (2), we get

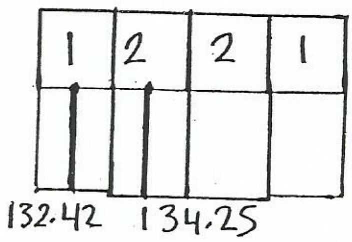
$P_{s1} = 132.42 \ kN/m^2$, $\quad P_{s2} = 134.25 \ kN/m^2$

- To calculate settlement, substitute in eqs (I), (II)

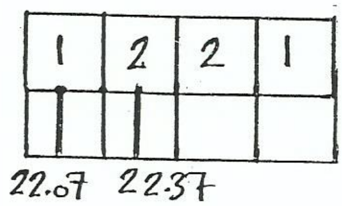
$\cdot \ \Delta_1 = \dfrac{132.42}{6000} = 0.022 \ m \ = 22.07 \ mm$

$\cdot \ \Delta_2 = \dfrac{134.25}{6000} = 0.02237 \ = 22.37 \ mm$

$\therefore \ \Delta_1 \simeq \Delta_2 \implies$ Rigid Footing



Contact stress
(kN/m²)

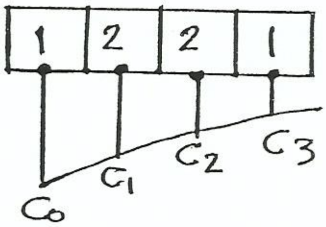


settlement
(mm)

(2) Ohde Assumption:-

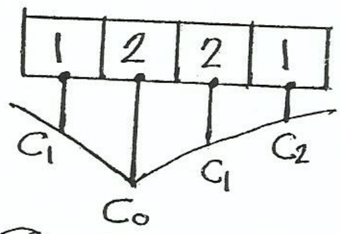
$$\bullet\ \Delta_i = \frac{a}{E_s} \sum_{i=1}^{n} C_i \times P_{s_i}$$

$$\Delta_1 = \frac{1}{4000}\left(1.33 P_{s_1} + 0.52 P_{s_2} + 0.3 P_{s_2} + 0.2 P_{s_1}\right)$$



$$\therefore\ \Delta_1 = \frac{1}{4000}\left(1.53 P_{s_1} + 0.82 P_{s_2}\right) \quad \text{--- (I)}$$

$$\therefore\ \Delta_2 = \frac{1}{4000}\left(0.52 P_{s_1} + 1.33 P_{s_2} + 0.52 P_{s_2} + 0.3 P_{s_1}\right)$$



$$\therefore\ \Delta_2 = \frac{1}{4000}\left(0.82 P_{s_1} + 1.85 P_{s_2}\right) \quad \text{--- (II)}$$

Final eqs:-

Substituting eqs (a, b), (I, II) into eq. (1), we get

$$\bullet\ \underbrace{5 \times (1.5 P_{s_1})}_{M2} = \frac{3.24 \times 10^6}{4000}\left(-0.71 P_{s_1} + 1.03 P_{s_2}\right)$$

$$\therefore\ \underline{582.6 P_{s_1} - 834.3 P_{s_2} = 0} \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$\therefore\ \underline{P_{s_1} + P_{s_2} = 266.67} \quad \text{------ (2)}$$

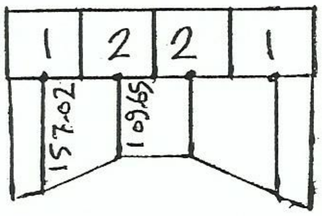
Solving eqs ① , ② , we get

$P_{S_1} = 157.02 \ KN/m^2$, $P_{S_2} = 109.65 \ KN/m^2$

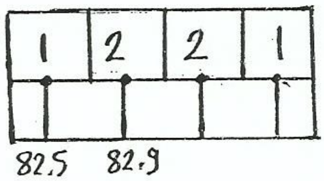
- To calculate settlement , substitute in eqs

$\Delta_1 = 0.0825 \ m = 8.25 \ cm$

$\Delta_2 = 0.0829 m = 8.29 \ cm$



Contact Stress
(KN/m²)



Settlement
(mm)

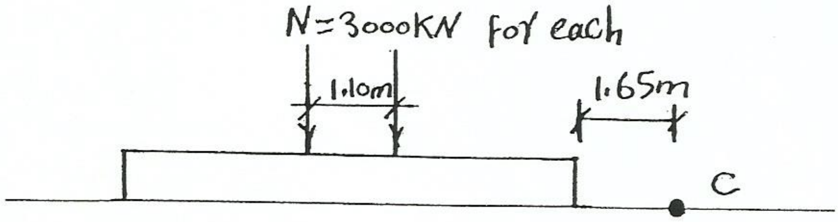
Example (2) :-

Compute and draw the distribution of contact stress below the R.C. isolated footing shown in Figure, considering the soil is elastic, homogeneous, isotropic, and semi-infinite space. The footing is (2.00X6.60m) and carries one column (120 X110 cm) which is subjected to normal compression load 6000 KN. The footing is divided into 6 elements. The column can be modelled as two loads spaced 1.10 m, 3000 KN for each. It is required also to :-

i) calculate the settlement at point (c).

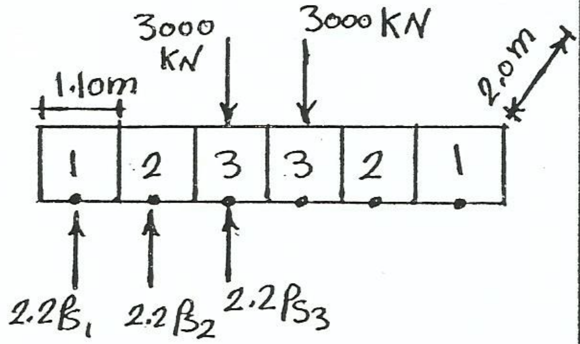
ii) calculate the maximum bending moment in the footing.

- ($C_0 = 1.5$, $C_1 = 0.65$, $C_2 = 0.35$, $C_3 = 0.25$, $C_4 = 0.18$, $C_5 = 0.13$, $C_6 = 0.09$)
- $E_c = 2 \times 10^7$ KPa , $E_s = 70$ MN/m^2
- Thickness of the footing = 130 Cm



Solution:-

- Assume contact stress (P_{si}) below each element
- P_i (element reaction) $= 1.1 * 2 * P_{si}$
  $= 2.2\ P_{si}$



(A) M - Ps eqs:-

$$M_1 = 0 \quad \text{-----} \quad (a)$$

$$M_2 = 2.2\,P_{s1} * 1.1 = 2.42\,P_{s1} \quad \text{-----} \quad (b)$$

$$M_3 = (2.2\,P_{s1} * 2.2) + (2.2\,P_{s1} * 1.1) = 4.84\,P_{s1} + 2.42\,P_{s2} \quad \text{--} \quad (c)$$

(B) M - Δ eqs:-

$$\frac{6 E_c I}{a^2} = \frac{6 \times 2.1 \times 10^7 \times \frac{2 \times (1.3)^3}{12}}{(1.10)^2} = 38.13 \times 10^6 \ KN$$

- Applying 4- Moment equation

Point(2):-

$$M_1 + 4 M_2 + M_3 = 38.13 \times 10^6 \left(-\Delta_1 + 2\Delta_2 - \Delta_3\right) \quad \text{---} \quad (1)$$

Point (3):-

$$M_2 + 5 M_3 = 38.13 \times 10^6 \left(-\Delta_2 + \Delta_3\right) \quad \text{-----} \quad (2)$$

c) S-Δ eqs:-

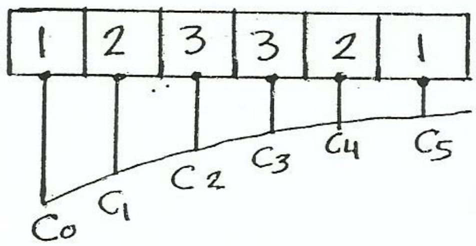
Using assumption of soil is elastic, homogeneous isotropic and semi-infinite

$$\Delta_i = \frac{a}{E_s} \sum_{i=1}^{n} C_i * P_{si}$$

$$\frac{a}{E_s} = \frac{1.1}{70\times10^3} = 1.57\times10^{-5} \ m^3/kN$$

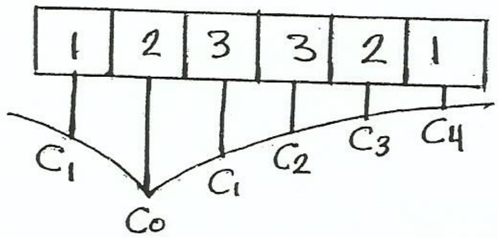
Point (1):-

$$\Delta_1 = 1.57\times10^{-5}\left[1.63P_{s1} + 0.83P_{s2} + 0.6P_{s3}\right] \quad \text{--- (I)}$$



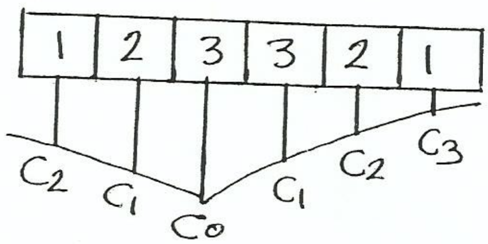
Point (2):-

$$\Delta_2 = 1.57\times10^{-5}\left[0.83\,P_{s1} + 1.75P_{s2} + P_{s3}\right] \quad \text{--- (II)}$$



Point (3):-

$$\Delta_3 = 1.57\times10^{-5}\left[0.6P_{s1} + P_{s2} + 2.15\,P_{s3}\right] \quad \text{--- (III)}$$



Final eqs :-

Substituting eqs [a, b, c], [I, II, III] into eqs ①, ②

eq. ① :-

$$0+4(2.42\,P_{S_1})+(4.84\,P_{S_1}+2.42\,P_{S_2})$$
$$=\underbrace{38.13*10^6*1.57*10^{-5}}_{598.641}\,(-0.57\,P_{S_1}+1.67\,P_{S_2}-0.75\,P_{S_3})$$

$$\therefore\ 355.745\,P_{S_1}-997.31\,P_{S_2}+448.98\,P_{S_3}=0 \quad ---\ ①$$

eq. ② :-

$$2.42\,P_{S_1}+5(4.84\,P_{S_1}+2.42\,P_{S_2})=598.641(-0.23\,P_{S_1}-0.75\,P_{S_2}+1.15\,P_{S_3})$$

$$\therefore\ 164.31\,P_{S_1}+461.1\,P_{S_2}-688.44\,P_{S_3}=0 \quad ---\ ②$$

eq. ③ :-

$$\Sigma F_y=0$$
$$\therefore\ 2*(2.2\,P_{S_1}+2.2\,P_{S_2}+2.2\,P_{S_3})=6000$$
$$\therefore\ P_{S_1}+P_{S_2}+P_{S_3}=1363.7 \quad ------\ ③$$

By solving eqs 1, 2, and 3, we get

$$P_{s1} = 580.724 \ kN/m^2$$

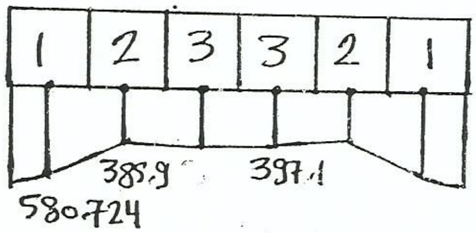
$$P_{s2} = 385.9 \ kN/m^2$$

$$P_{s3} = 397.1 \ kN/m^2$$

Check:-

$$P_{s\,avg.} = 454.5 \ kN/m^2$$

$$= \underbrace{q_{act.}}_{\text{Contact stress}} = \frac{\overset{\text{col load}}{6000}}{\underbrace{2 \times 6.60}_{\text{Footing Area}}} = 454.5 \ kN/m^2$$



Contact stress (kN/m^2)

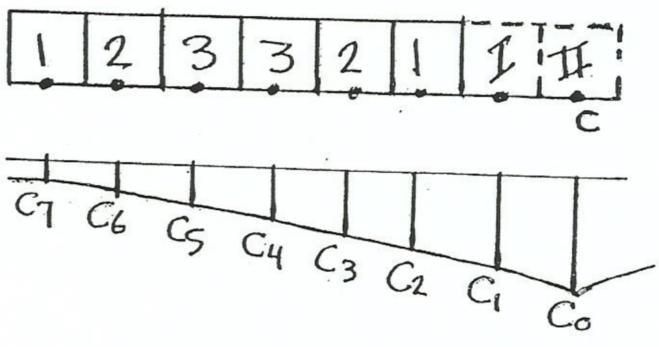
Settlement at Point (c):-

C_7 is unknown

$$\because C_i = \frac{C_0}{1 + k_1 (i)^{k_2}}$$

$$\because C_1 = \frac{1.5}{1 + k_1 (1)^{k_2}} = 0.65$$

$\hookrightarrow k_1 = 1.3$



$$\because C_2 = \frac{1.5}{1 + \underset{1.3}{k_1} (2)^{k_2}} = 0.35$$

$\hookrightarrow k_2 = 1.337$

$\therefore\ C_7 = \dfrac{1.5}{1+1.3(7)^{1.337}} = 0.08 \quad (\text{check} < C_6)$

$\therefore\ \Delta c = 1.57\times10^{-5}\left[C_0 * P_{S_I} + C_1 P_{S_{II}} + C_2 P_{S_1} + C_3 P_{S_2} + C_4 P_{S_3} + C_5 P_{S_3} + C_6 P_{S_2} + C_7 P_{S_1}\right]$

$\therefore\ \Delta c = 1.57\times10^{-5}\left[1.5 * 0 + 0.65\times0 + 0.43 P_{S_1} + 0.34 P_{S_2} + 0.31 P_{S_3}\right]$

$= 1.57 * 10^{-5}\left[0.43\times580.724 + 0.34\times385.9 + 0.31\times397.1\right]$

$= 7.913\times10^{-3}\ m$

$= \underline{\underline{7.913\ mm}}$

Calculation of Max. B.M. in footing:-

- $M_1 = 0$
- $M_2 = 2.42\ P_{S_1} = 2.42\times580.724 = 1405.35\ KN.m.$
- $M_3 = 4.84\ P_{S_1} + 2.42\ P_{S_2}$

  $= 4.84\times580.724 + 2.42\times385.9$

  $= 3744.58\ KN.m.$

$\therefore\ \boxed{M_{max.} = M_3 = 3744.58\ KN.m.}$

Example ③:-

The R.C. strip footing shown in Figure is 2.00 X 6.00 m and carries two columns 3.00 meters center to center. Each column is subjected to: $N = 1000$ kN and $M = 50$ kN.m at opposite directions. The footing is divided into 6 elements.

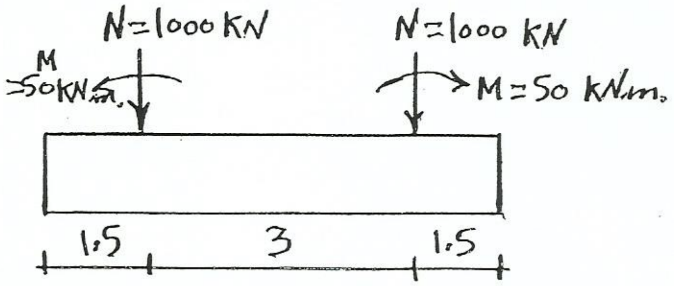
It is required to:-

a) Determine the contact stress below the footing on the basis of Winkler assumption ($K_{s0} = 2500$ kN/m^3, $t_{Footing} = 120$ cm)

b) Determine the contact stress under the footing assuming the soil to be elastic, homogeneous, isotropic, and semi-infinite.

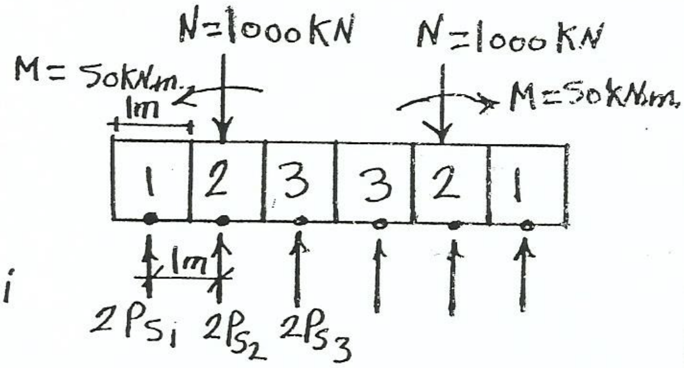
- ($C_0 = 1.33$, $C_1 = 0.52$, $C_2 = 0.30$, $C_3 = 0.20$, $C_4 = 0.16$, $C_5 = 0.12$),
- $E_c = 2000$ kN/cm^2
- $E_s = 1200$ kN/m^2, thickness of footing $= 120$ cm

c) Compare between the two solutions and explain the difference.



Solution:-

- Assume contact stress (P_{si}) under each element



- P_i (element reaction) $= a \times b \times P_{si}$
  $= 2\,P_{si}$

(A) M-Ps eqs:-

$$M_1 = 0 \quad \text{------} \quad (a)$$

$$M_2 = 2\,P_{s1} \quad \text{------} \quad (b)$$

$$M_3 = 4\,P_{s1} + 2\,P_{s2} - 1050 \quad \text{------} \quad (c)$$

(B) M-Δ eqs:-

$$\frac{6\,E_c\,I}{(a)^2} = \frac{6 \times 2 \times 10^7 \times 2 \times \frac{(1.2)^3}{12}}{(1)^2} = 34.56 \times 10^6 \text{ KN}$$

- Applying 4-Moment equation at Pts 2, and 3

Point 2:-

$$M_1 + 4\,M_2 + M_3 = 34.56 \times 10^6 \left(-\Delta_1 + 2\Delta_2 - \Delta_3\right) \quad \text{----} \quad (1)$$

Point 3:-

$$M_2 + 5\,M_3 = 34.56 \times 10^6 \left(-\Delta_2 + \Delta_3\right) \quad \text{----} \quad (2)$$

(c) $\Delta - P_s$ eqs:-

1) Winkler assumption:-

$$\Delta_i = \frac{P_{si}}{K_{so}}$$

$$\therefore \quad \Delta_1 = \frac{P_{s1}}{2500} \quad \text{----} \quad (I)$$

$$\Delta_2 = \frac{P_{s2}}{2500} \quad \text{-----} \quad (II)$$

$$\Delta_3 = \frac{P_{s3}}{2500} \quad \text{--------} \quad (III)$$

Final eqs:-

Substituting eqs (a, b, c), (I, II, III) into eqs ①, ②, we get:-

eq. ①:-

$$0 + (4 \times 2\, P_{s1}) + (4P_{s1} + 2P_{s2} - 1050) = \frac{34.56 \times 10^6}{2500}\left(-P_{s1} + P_{s2} - P_{s3}\right)$$

$$\therefore \quad 13836\, P_{s1} - 27646\, P_{s2} + 13824\, P_{s3} = 1050 \quad \text{----} \quad (1')$$

eq. ②:-

$$(2\, P_{s1}) + (20\, P_{s1} + 10\, P_{s2} - 5250) = \frac{34.56 \times 10^6}{2500}\left(-P_{s2} + P_{s3}\right)$$

$$22\, P_{s1} + 13834\, P_{s2} - 13824\, P_{s3} = 5250 \quad \text{---} \quad (2)$$

eq. (3) :-

$$\Sigma F_y = 0$$

$$\therefore 2\left[2P_{s1} + 2P_{s2} + 2P_{s3}\right] = 2000$$

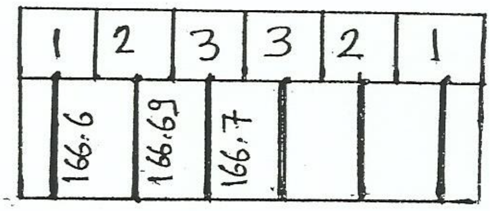
$$\therefore \underline{P_{s1} + P_{s2} + P_{s3} = 500} \quad \text{------------} \quad (3)$$

Solving eqs (1), (2), and (3), we get :-

$P_{s1} = 166.6 \ kN/m^2$

$P_{s2} = 166.69 \ kN/m^2$

$P_{s3} = 166.7 \ kN/m^2$ ..



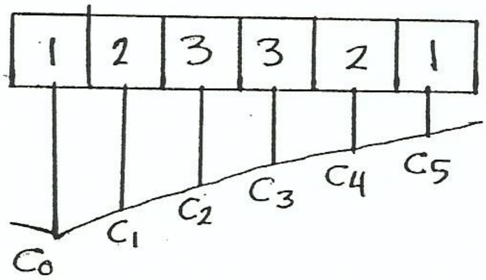
Contact stress
(KPa)

2) ohde Assumption:-

$$\Delta_i = \frac{a}{E_s} \sum_{i=1}^{n} C_i \times P_{si}$$

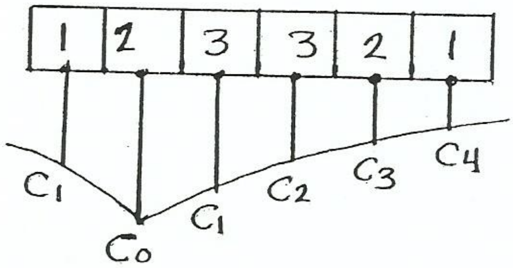
Point(1):-

$$\Delta_1 = \frac{1}{1200}\left(1.45\, P_{s1} + 0.68\, P_{s2} + 0.5\, P_{s3}\right) \quad \text{--- (I)}$$



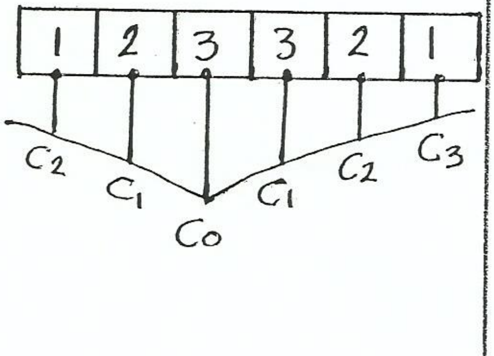
Point (2):-

$$\Delta_2 = \frac{1}{1200}\left(0.68\, P_{s1} + 1.53\, P_{s2} + 0.82\, P_{s3}\right) \quad \text{--- (II)}$$



Point (3):-

$$\Delta_3 = \frac{1}{1200}\left(0.50\, P_{s1} + 0.82\, P_{s2} + 1.85\, P_{s3}\right)$$



Final eqs :-

substituting eqs (a, b, c), (I, II, III) into eqs ①, ②, we get :-

eq ① :-

$$0 + (4\times 2\,P_{S_1}) + (4P_{S_1} + 2P_{S_2} - 1050) = \frac{34.56\times10^6}{1200}\left(-0.59\,P_{S_1} + 1.56\,P_{S_2} - 0.71\,P_{S_3}\right)$$

$$\therefore\ 16997.2\,P_{S_1} - 44908.03\,P_{S_2} + 20439.82\,P_{S_3} = 1050 \quad ----\ ①$$

eq ② :-

$$2P_{S_1} + (20P_{S_1} + 10P_{S_2} - 5250) = \frac{34.56\times10^6}{1200}\left(-0.18\,P_{S_1} + 0.71\,P_{S_2} + 1.03\,P_{S_3}\right)$$

$$5203.93\,P_{S_1} + 20449.82\,P_{S_2} - 29652.13\,P_{S_3} = 5250 \quad ----\ ②$$

eq. ③ :-

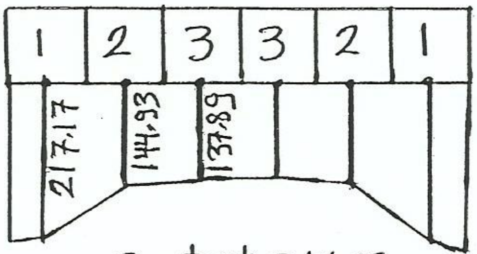
$$P_{S_1} + P_{S_2} + P_{S_3} = 500 \quad ----\ ③$$

- Solving eqs ①, ②, ③

$P_{S_1} = 217.17\ KN/m^2$

$P_{S_2} = 144.93\ KN/m^2$

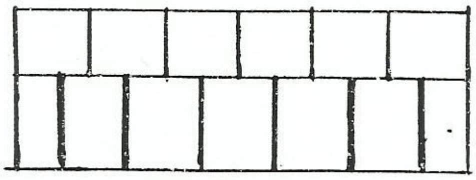
$P_{S_3} = 137.89\ KN/m^2$



Contact stress (KPa)

Comparison between the two Solutions:-

Winkler assumption

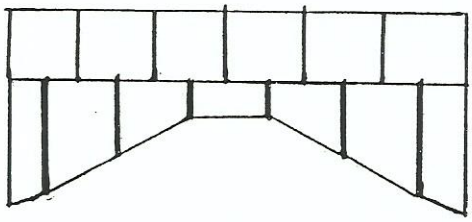


1) يتم اهمال الـ shear stiffness بين الـ elements ، وبالتالى تتغير قيمة الهبوط فجائياً من element لآخر.

2) يتم اهمال خواص التربة (E_s) عند حساب الهبوط.

- $\Delta_i = \dfrac{P_{si}}{K_{s0}}$

Ohde assumption



1) يتم أخذ الـ shear stiffness بين الـ elements في الإعتبار، و الهبوط أسفل كل element يتأثر بالإجهاد المؤثر على الـ elements المجاورة ، و بالتالى يكون التغير في قيمة الهبوط تدريجي.

2) يتم أخذ خواص التربة (E_s) في الإعتبار عند حساب الهبوط.

- $\Delta_i = \dfrac{a}{E_s} \sum_{i=1}^{n} C_i * P_{si}$

- الفرق بين الحلين أن الإجهاد المحسوب بإستخدام طريقة Winkler في حالة (Rigid Footing) هو متوسط قيمة الإجهاد أسفل القاعدة.

Example (4):-

It is decided to design R.C. strip footing considering the soil elasticity and homogeneity. The footing is 1.20×8.00 meters and carries a masonry wall. The total load on wall is 250 kN/m'. The footing is rested on an over-consolidated clay deposit 10.00 meters thick (from level 0.00 to level −10.00), overlying a layer of sand stone. The elastic modulus of clay increases from 10 MN/m² at F.L. (−2.00m) to 26 MN/m² at the end of the deposit. The footing is divided into five elements. It is required to:-

a) Compute and draw the distribution of contact stress below the footing assuming the soil is elastic, homogeneous, isotropic, and semi-infinite.

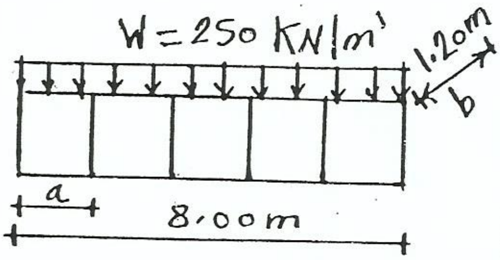
$$\left(C_o = 1.60 \,, \quad C_i = \frac{C_o}{1 + 1.2(i)^{1.4}} \right)$$

Data:- $E_c = 2 \times 10^7$ kN/m², thickness of footing = 100 cm

b) Sketch the expected contact stress distribution considering Winkler assumption.

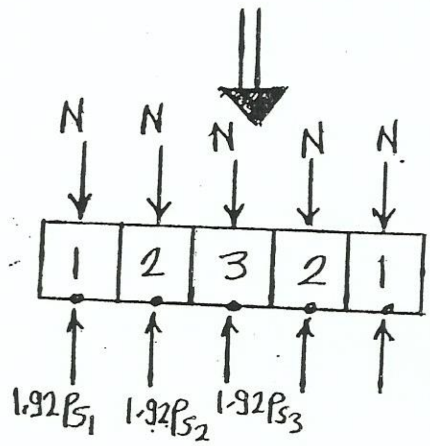
Solution :-

$a = 8/5 = 1.60 m$



Loads :-

$N = W * a$
$= 250 * 1.60$
$= 400 \ KN$



Reactions :-

- Assume contact stress (P_{si}) under each element
- P_i (Reaction on each element) $= a \times b \times P_{si}$
$= 1.6 \times 1.2 \times P_{si}$
$= 1.92 \ P_{si}$

A) M-Ps eqs :-

$M_1 = 0$

$M_2 = (1.92 P_{s1} * 1.60) - (400 * 1.60) = 3.072 P_{s1} - 640$ --- ⓐ

$M_3 = (1.92 P_{s1} * 3.20) + (1.92 P_{s2} * 1.60) - (400 * 3.20) - (400 * 1.60)$

$= 6.144 P_{s1} + 3.072 P_{s2} - 1920$ ------ ⓑ

(B) M-Δ eqs:-

$$\frac{6E_c I}{a^2} = \frac{6*2*10^7*\frac{1.2*(1)^3}{12}}{(1.6)^2} = 4.6875*10^6 \ kN$$

- Applying 4-Moment equation

Point (2):-

$$M_1 + 4M_2 + M_3 = 4.6875*10^6\left(-\Delta_1 + 2\Delta_2 - \Delta_3\right) \quad \text{------} \ (1)$$

Point (3):-

$$M_2 + 5M_3 = 4.6875*10^6\left(-\Delta_2 + \Delta_3\right) \quad \text{------} \ (2)$$

(C) Δ-Ps eqs:-

- $$\Delta_i = \frac{a}{E_{S\,avg.}} * \sum_{i=1}^{n} C_i * P_{Si}$$

$$* \ E_{S_{avg.}} = \frac{10+26}{2} = 18 \ MN/m^2 = 18*10^3 \ kN/m^2$$

$$\therefore \frac{a}{E_{S\,avg.}} = \frac{1.60}{18*10^3} = 8.89*10^{-5} \quad m^3/kN$$

$$\therefore C_i = \frac{C_o}{1+1.2(i)^{1.40}} \qquad (C_o = 1.60)$$

$$\therefore C_1 = \frac{1.60}{1+1.2(1)^{1.4}} = 0.727$$

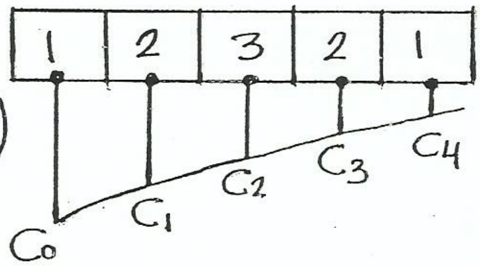
$$C_2 = \frac{1.60}{1+1.2(2)^{1.4}} = 0.384$$

$$C_3 = \frac{1.60}{1+1.2(3)^{1.4}} = 0.243$$

$$C_4 = \frac{1.60}{1+1.2(4)^{1.4}} = 0.17$$

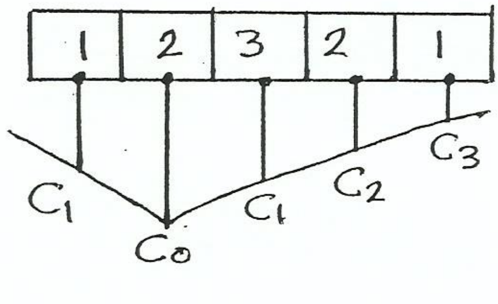
Point (1) :-

$$\Delta_1 = 8.89 * 10^{-5}\left(1.77 P_{S_1} + 0.97 P_{S_2} + 0.384 P_{S_3}\right) \quad \text{------ (I)}$$



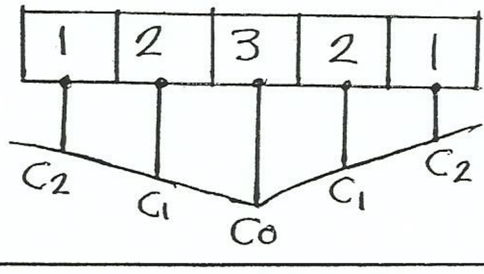
Point (2) :-

$$\Delta_2 = 8.89 * 10^{-5}\left(0.97 P_{S_1} + 1.984 P_{S_2} + 0.727 P_{S_3}\right) \quad \text{------ (II)}$$



Point (3) :-

$$\Delta_3 = 8.89 * 10^{-5}\left(0.768 P_{S_1} + 1.454 P_{S_2} + 1.60 P_{S_3}\right) \quad \text{------ (III)}$$



Final eqs :-

eq. (1) :-

$$0 + (4 * 3.072\, P_{S1} - 4 * 640) + (6.144\, P_{S1} + 3.072\, P_{S2} - 1920)$$

$$= \underbrace{[4.6875 * 10^{6} * 8.89 * 10^{-5}]}_{416.72} * (-0.598\, P_{S1} + 1.544\, P_{S2} - 0.53\, P_{S3})$$

$$\therefore\ 267.63\, P_{S1} - 640.34\, P_{S2} + 220.86\, P_{S3} = 4480 \quad \text{--- (1)}$$

eq. (2) :-

$$(3.072\, P_{S1} - 640) + (3.072\, P_{S1} + 15.36\, P_{S2} - 9600)$$

$$= 416.72\, (-0.202\, P_{S1} - 0.53\, P_{S2} + 0.873\, P_{S3})$$

$$\therefore\ 117.97\, P_{S1} + 236.22\, P_{S2} - 363.8\, P_{S3} = 10240 \quad \text{--- (2)}$$

eq. (3) :-

$$\Sigma F_y = 0$$

$$2 * (1.92\, P_{S1} + 1.92\, P_{S2}) + 1.92\, P_{S3} = 5 * 400$$

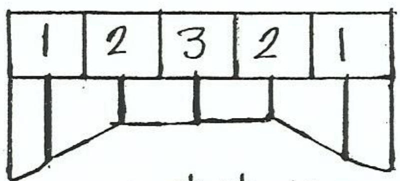
$$\therefore\ 3.84\, P_{S1} + 3.84\, P_{S2} + 1.92\, P_{S3} = 2000 \quad \text{--- (3)}$$

- Solving eqs 1, 2, 3, we get :-

$P_{S1} = 272.65\ kN/m^2$

$P_{S2} = 164.60\ kN/m^2$

$P_{S3} = 167.15\ kN/m^2$



Contact stress (KPa)

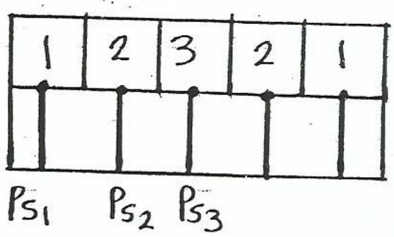
b) Expected Contact stress from winkler:-

- Based on ohde assumption results, the footing is rigid

$$\therefore P_{s_{avg.}} = \frac{2\,(272.65 + 164.60) + 167.15}{5} = 208.3 \ kN/m^2$$

- For winkler assumption results

$$P_{s_1} \simeq P_{s_2} \simeq P_{s_3} \simeq P_{s_{avg.}}$$



Uniform contact stress distribution

Example ⑤ :-

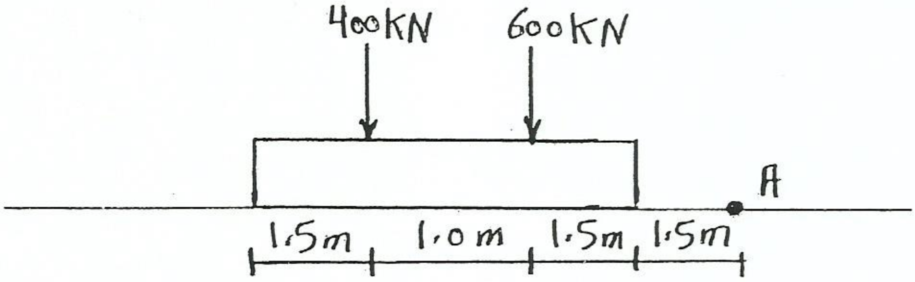
The shown R.C. footing is 1.20*4.00m, with thickness = 60 cm, and carries two columns spaced 1.0m center to center. Divide the footing into 4 elements, and write the equations required to calculate the soil reactions below the footing, assuming the soil to be elastic, homogeneous, isotropic, and semi-infinite.

($C_0 = 1.33$, $C_1 = 0.52$, $C_2 = 0.3$, $C_3 = 0.2$)

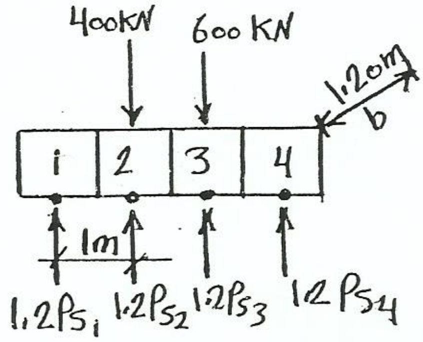
1- Construct the matrix form for the equations without solving them.

$E_c = 2*10^7\ KN/m^2$, $E_s = 6000\ KN/m^2$

2- Assuming average contact stress below the previous footing, it is required to calculate the expected settlement at point (A).



Solution:-



- Assume contact stress (P_{si}) below each element

- P_i (Reaction) $= 1 * 1.2 * P_{si}$
  $= 1.2\, P_{si}$

A) M–Ps eqs:-

$M_1 = 0$ ——— ⓐ

$M_2 = 1.2\, P_{s1}$ ——— ⓑ

$M_3 = 2.4\, P_{s1} + 1.2\, P_{s2} - 400$ ——— ⓒ

$M_4 = 3.6\, P_{s1} + 2.4\, P_{s2} + 1.2\, P_{s3} - (400*2) - (600*1)$

$= 3.6\, P_{s1} + 2.4\, P_{s2} + 1.2\, P_{s3} - 1400$ --- ⓓ

B) M–Δ eqs:-

$$\frac{6 E_c I}{a^2} = \frac{6 * 2 * 10^7 * 1.2 * (0.6)^3 / 12}{(1)^2} = 2.592 * 10^6 \text{ KN}$$

- Applying 4- Moment equation

Point (2):-

$M_1 + 4M_2 + M_3 = 2.592 * 10^6 \left[-\Delta_1 + 2\Delta_2 - \Delta_3\right]$ --- ①

Point (3):-

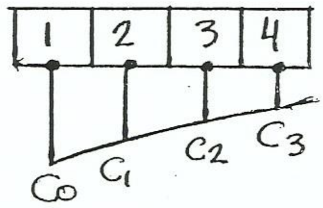
$$M_2 + 4M_3 + M_4 = 2.592 * 10^6 \left[-\Delta_2 + 2\Delta_3 - \Delta_4 \right] \quad \text{——— (2)}$$

c) $\Delta - P_s$ eqs:-

$$\Delta_i = \frac{a}{E_s} \sum C_i * P_{s_i}$$

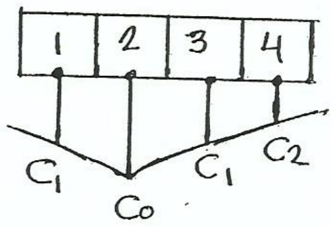
Point (1):-

$$\Delta_1 = \frac{1}{6000} \left(1.33 P_{s_1} + 0.52 P_{s_2} + 0.3 P_{s_3} + 0.2 P_{s_4} \right) \quad \text{——— (I)}$$



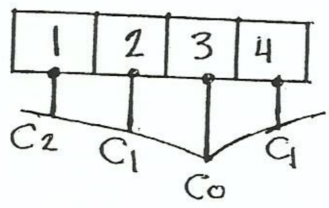
Point (2):-

$$\Delta_2 = \frac{1}{6000} \left(0.52 P_{s_1} + 1.33 P_{s_2} + 0.52 P_{s_3} + 0.3 P_{s_4} \right) \quad \text{——— (II)}$$



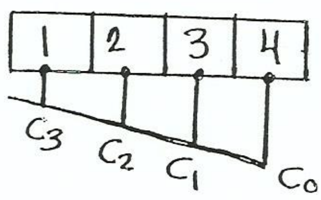
Point (3):-

$$\Delta_3 = \frac{1}{6000} \left(0.3 P_{s_1} + 0.52 P_{s_2} + 1.33 P_{s_3} + 0.52 P_{s_4} \right) \quad \text{——— (III)}$$



Point (4):-

$$\Delta_4 = \frac{1}{6000} \left(0.2 P_{s_1} + 0.3 P_{s_2} + 0.52 P_{s_3} + 1.33 P_{s_4} \right) \quad \text{——— (IV)}$$



Final equations:-

substituting eqs ⓐ, ⓑ, ⓒ, ⓓ and I, II, III, IV into eqs ①, ②

eq. ①

$$0 + (4 * 1.2\, P_{S_1}) + (2.4\, P_{S_1} + 1.2\, P_{S_2} - 400)$$
$$= \frac{2.592 * 10^6}{6000}\left[-0.59\, P_{S_1} + 1.62\, P_{S_2} - 0.59\, P_{S_3} - 0.12\, P_{S_4}\right]$$

$$\therefore\ 262.08\, P_{S_1} - 698.64\, P_{S_2} + 254.88\, P_{S_3} + 51.84\, P_{S_4} = 400 \quad \text{①}$$

eq. ②

$$1.2\, P_{S_1} + 4(2.4\, P_{S_1} + 1.2\, P_{S_2} - 400) + (3.6\, P_{S_1} + 2.4\, P_{S_2} + 1.2\, P_{S_3} - 1400)$$
$$= \frac{2.592 * 10^6}{6000}\left[-0.12\, P_{S_1} - 0.59\, P_{S_2} + 1.62\, P_{S_3} - 0.59\, P_{S_4}\right]$$

$$\therefore\ 66.24\, P_{S_1} + 262.08\, P_{S_2} - 698.64\, P_{S_3} + 254.88\, P_{S_4} = 3000 \quad \text{②}$$

eq. ③:-

$$\Sigma F_y = 0$$
$$1.2\,(P_{S_1} + P_{S_2} + P_{S_3} + P_{S_4}) = 400 + 600$$
$$\therefore\ P_{S_1} + P_{S_2} + P_{S_3} + P_{S_4} = 833.3 \quad \text{③}$$

eq. ④:-

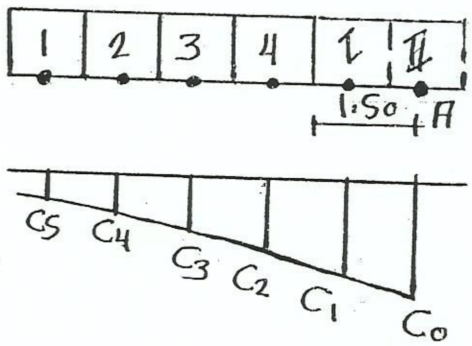
$$M_4 = \text{zero} \Longrightarrow 3.6\, P_{S_1} + 2.4\, P_{S_2} + 1.2\, P_{S_3} = 1400 \quad \text{④}$$

Final eqs in Matrix form:-

$$\begin{bmatrix} P_{s_1} \\ P_{s_2} \\ P_{s_3} \\ P_{s_4} \end{bmatrix} * \begin{bmatrix} 262.08 & -698.64 & 254.88 & 51.84 \\ 66.24 & 262.08 & -698.64 & 254.88 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 3.6 & 2.4 & 1.2 & 0 \end{bmatrix} = \begin{bmatrix} 400 \\ 3000 \\ 833.3 \\ 1400 \end{bmatrix}$$

Settlement at Point (A)

- Assuming Average Contact stress below footing



$\therefore P_{s_1} = P_{s_2} = P_{s_3} = P_{s_4} = P_{s_{avg.}}$

$$P_{s_{avg.}} = \frac{400+600}{4 * 1.2} = 208.33 \text{ KN/m}^2$$

- C_4, C_5 are unknow

$$\therefore C_i = \frac{C_o}{1 + k_i (i)^{k_2}}$$

$$\therefore C_1 = \frac{1.33}{1 + K_1 (1)^{K_2}} = 0.52 \longrightarrow K_1 = 1.56$$

$$\therefore C_2 = \frac{1.33}{1 + 1.56 (2)^{K_2}} = 0.3 \longrightarrow K_2 = 1.13$$

$$\therefore\ C_4 = \frac{1.33}{1+1.56(4)^{1.13}} = 0.157$$

$$C_5 = \frac{1.33}{1+1.56(5)^{1.13}} = 0.125$$

$$\therefore\ \Delta_A = \frac{1}{6000}\left[C_0 * 0 + C_1 * 0 + 208.33\left(\underset{C_2}{0.3} + \underset{C_3}{0.2} + \underset{C_4}{0.157} + \underset{C_5}{0.125}\right)\right]$$

$$= 0.027\,m$$

$$\therefore\ \underline{\Delta_A = 2.7\ cm}$$